Indian Statistical Institute
Mid-Semestral Examination 2003-2004
B.Math I Year II Semester
Mathematical Analysis II (Supplementary)

Time: 3 hrs Date:08-03-04 Max. Marks : 35

Answer all the questions.

1. Let (X_1, d_1), (X_2, d_2) be metric spaces, $Y = X_1 \times X_2$ with metric d given by $d((x_1,\ x_2), (a_1,\ a_2)) = \sqrt{[d_1(x_1,\ a_1)]^2 + [d_2(x_2,\ a_2)]^2}$

   a) If X_1 is not connected, show that $Y$is not connected. [3]

   b) Let $S = \{(x,\ y) \in R^2 \quad : \quad x \neq 0\}$. Show that S is not connected [2]

2. Let X_1, X_2, Y be as m be as in Question 1. If X_1, X_2 satisfy Bolzano property [Recall: A metric space S has Bolzano property if every sequence has a convergent subsequence] [3]

3. Let $\varphi : [0, 1] \to R$ be any continuous function with $\varphi(0) = 0$. Define $\psi : [0, 1] \to R$ by $\psi(x) = \varphi(x)sin(\frac{1}{x})$ for $x \neq 0$ and $\psi(0) = \varphi(0)$. Show that ψ is continuous function on $[0, 1]$ [4]

4. Let $f : [1, \infty] \to R$ be given by $f(x) = x^{\frac{3}{2}}$. Show that f is not uniformly continuous. [5]

5. a) Let (X, d) be a metric space. Show that in a triangle the difference between any two sides is less than the third side i.e., $|d(x, y) - d(x, z)| \leq d(y, z)$ for all x, y, z in X [3]

   b) Let (X, d) be a connected metric space such that $\sup\{d(x_0, y) : y \in X\} = \infty$ for some x_0. Show that $\{d(x_0, y) : y \in x\} = [0, \infty]$ [3]

   c) Let B be any nonempty, non compact subset of R. Show that there exist a continuous $f : B \to R$ such that f is not bounded. [4]

6. Let $X = M_{2 \in 2}(R)$ be the space of all $2 \in 2$ matrices with real entries. Define $\left\| \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \right\| = (\sum\limits_{i,j} \ a_{ij}^2)^{\frac{1}{2}}$. Fix A_0 in X. Define $L : X \to X$ by $L(B) = A_0 B$. Find the derivative of L. [8]